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SCIMATH302 - Advanced Mathematics; Abstract Algebra

Fall 2019 Course Manual

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Contents

Course code:	SCIMATH302
Course title:	Advanced Mathematics; Abstract Algebra
Semester:	Fall 2019
Classroom no:	T.B.A.
Class times:	MON 13:45 - 15:45 and THU 08:45 - 10:45
Instructor:	Dr. L.R. (Richard) van den Doel
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Office hours:	MON to THU 09:00 - 17:00

1 Track information

a) Prerequisites for this course: (SCIMATH101 and SCIMATH202) or (SCIMATH101 and SCIMATH203).
b) Other courses which are relevant to this course - *e.g.* as part of a minor: SCIMATH101, SCIMATH202, SCIMATH203, SCIMATH301.

For further information about the track, please see the track document available on the UCR intranet.

2 Course Description

This course is an introduction to the fascinating field of Abstract Algebra; it will introduce some of the fundamental algebraic systems that are both interesting and of wide use. The aim of this course is to arrive at some significant results in each of these systems.
The starting point of this course is the following: consider a collection of objects and assume that it is possible to combine the elements of this set to obtain again the elements of this set; combining two elements of the set is a binary operation. Then condition or regulate the nature of the set by imposing rules on how these operations behave on the set; these axioms define the particular structure on the set. In the course of time, it was noticed that there are many concrete mathematical systems that satisfy these axioms. This course studies some of the basic axiomatic algebraic systems: groups, rings and fields.
A **group** can be described by a closed set of elements equipped with a single binary operation (the operation is referred to as "multiplication", but it is an arbitrary binary operation!); this operation is associative. The set has an identity element and an inverse element for each element of the set. A **ring** is a closed set of elements under two binary operations (multiplication and addition); both binary operations are associative, but only addition is commutative. Furthermore, the two operations are distributive and there is a zero element and an additive inverse element for each element of the set. A ring is a

group under addition, but not under multiplication. Finally, a **field** is a commutative (under multiplication) ring in which every nonzero element has a multiplicative inverse; a field allows for the operations addition, subtraction, multiplication and division. A field can be considered as a group under addition and a group under multiplication.

3 Study load

This course earns students four UCR credits (equivalent to 7.5 EC). The class meets twice a week for two hours. Preparation time is approximately 10 hours per week.

4 Course materials

The following textbook is required for this course:

Abstract Algebra,
3rd edition (1999),
written by **I.N. Herstein** and
published by **John Wiley and Sons, Inc.**,
ISBN-number is 0-471-36879-2.

Compared to the other textbooks used in the Mathematics track, this textbook is small and contains almost no equations and no figures, tables or graphs. On the other hand, the goal was never to discuss each and every chapter of the other textbooks, whereas in this course the aim is to work through the entire booklet, except for the final chapter! The textbook by Herstein consists of six chapters: the first chapter is an introductory chapter. Most of the material is already known or discussed in other courses. The second chapter introduces the concept of a group. This chapter is the largest, most important and most challenging chapter of the course! Chapter three discusses the symmetric group. Chapters four and five introduce the concepts of rings and fields, respectively. Finally, the last chapter discusses a number of special topics.

4.1 Other Textbooks and resources

Numerous other textbooks providing an introduction to Abstract Algebra are available. See the short-list below:

- **A First Course In Abstract Algebra** by John B. Fraleigh, published by Addison Wesley.
- **Contemporary Abstract Algebra** by Joseph A. Gallian, published by Brooks / Cole.
- **Topics in Algebra** - Big Herstein - by I.N. Herstein, published by John Wiley and Sons.
- **An Introduction to the Theory of Groups** by Joseph J. Rotman, number 148 in the Graduate Texts in Mathematics Series published by Springer.
- **Galois Theory** by Ian Stewart, published by Chapman & Hall / CRC Mathematics

Furthermore, many explanations, examples, problems and solutions can be found on internet, for instance, the online textbook **Abstract Algebra: The Basic Graduate Year** at http://www.math.uiuc.edu/~r-ash/Algebra.html. Note that different textbooks use different notations for different quantities.

5 Course organization and requirements

a) General format of class meetings: all classes start with summarizing the most important results from the previous class(es). Student get the opportunity to ask questions regarding the material discussed earlier in class or regarding homework assignments. The remainder of the class is used to discuss new material. The classes prior to an exam are meant for revision; students can ask questions regarding the material for the upcoming exam, including problems from old exams.
b) Students are expected to come to class prepared and to participate actively during discussions and problem solving. Students are strongly encouraged to work together on the homework assignments.
c) Rules for missing classes and deadlines: the rules of the UCR Student Handbook apply. Homework assignments submitted after the deadline will not be corrected and graded. Students missing an exam are expected to notify the instructor before the actual exam and they are expected to retake the exam as soon as possible, preferably in the same week as the scheduled exam.
d) Procedures for communication and use of Moodle: the instructor has an open door policy and tries to be available anytime for discussing any issue in the course. Moodle will be used to make additional material available to students.

This course is subject to UCR academic rules and procedures. Both students and instructors are required to know and follow these rules and procedures.

6 Assessment

The following assessments are scheduled for this course:

6.1 Homework Series

- the main purpose of homework series is to gain experience with solving typical kinds of problems in the field of abstract algebra.
- Homework series will prepare students for the exams in this course.
- The homework series are the minimal set of problems students should try to solve. Numerous other problems will be solved during class, but students are also encouraged to solve other problems themselves in order to gain this experience.
- Students are encouraged to collaborate on the homework assignments.
- Students are encouraged to use as many other sources as possible with the exception, of course, of available solutions of these problems.
- An important form of feedback are the detailed step-by-step solutions of these homework assignments provided by the instructor after submitting the homework series fro grading.
- Students are expected to submit a total of six homework series.
- The deadline for submitting homework series can be found in Sec.7 Course Schedule.
- Homework series submitted after the deadline will not be corrected and graded.
- In case a student requests and gets an extension for submitting homework series after the deadline, the student is expected to submit the work already completed by the deadline for grading and to submit the remainder of the homework assignment by the new agreed deadline.

- Each homework series consists of 12 textbook problems.
- The exact textbook problems per homework series will be announced on Moodle.
- Homework assignments submitted for grading must contain explanations of the problem-solving strategy applied and must show the various steps and derivations of the problem-solving strategy.
- Students are encouraged to write the solutions of the homework assignments in LaTeX.
- Homework must be printed or written on plain A4 paper. The work must be stapled and most importantly, it must be legible in case the solutions are handwritten!
- The average grade of the homework series contributes 15% to the final grade.
- Students can earn maximally 4 points per problem:

4 points	**3 points**	**2 points**	**1 points**	**0 points**
Solution of problem is correct, clearly presented step-by-step and in a logical way, is complete and does not contain any flaws.	Solution of problem is correct, clearly presented in a logical way, is nearly complete and contains at most minor flaws.	Part of solution of problem is correct, presentation is not sufficiently detailed and not logical, is incomplete and contains at major flaws.	Solution of problem is incorrect but contains some correct steps, presentation is not sufficiently detailed and not logical, is incomplete and contains major flaw.	The solution of problem is completely wrong or missing, there are no correct steps given.

6.2 Writing Assignment

One of the results discussed in the course is the Fundamental Theorem on Finite Abelian Groups at the end of Ch. 2. The aim of this writing assignment is to put this fundamental theorem to practice and write a short paper on this. Students choose a well-defined abelian group of some interesting order and demonstrate how this abelian group can be written as the direct product of cyclic groups.

- While the aim of the homework series is to find solutions to typical kinds of abstract algebra problems, the aims of the writing assignment also include presenting the solution of a somewhat larger problem in the form of a short paper.
- This paper consists of an introduction, a main part discussing the solutions steps of the problem in detail and a conclusions and discussion section. The writing assignment has to be written in LaTeX.
- The writing assignment also includes some work that requires the use of *Mathematica*.
- Students can submit a draft version of the writing assignment before submitting the final assignment.
- The writing assignment contributes 10% to the final grade. Students will receive feedback on this writing assignment.
- Sec. 7 Course Schedule shows the deadline for submitting the writing assignment.
- The writing assignment will be assessed using the table below:

- **title**: 0–5 points: the title of the paper is presented in the following form:
  TITLE
  STUDENT NAME
  DATE
  This writing assignment is written in partial fulfillment for the UCR course
  SCIMATH302 - Advanced Mathematics; Abstract Algebra
  instructor: Dr. L.R. van den Doel
  in the Fall 2019 semester.
  Missing parts in the the title results in a deduction of points.
- **introduction**: 0–15 points: the introduction carefully explains the topic of the writing assignment and outlines what the writing assignment will discuss?
- **main text**: 0–60 points: the main text presents the fundamental theorem of finite abelian groups, but NOT a full step-by-step proof of the theorem. The key ideas and main steps of the proof are given. The student is encouraged to present part(s) of the proof in some detail. The following aspects are important in the main part of the paper:
  * Quantities and symbols are correctly introduced? Max. 2 point deduction if incorrect.
  * Equations are correctly used? Max. 5 point deduction if incorrect.
  * Tables are correctly used? Max. 2 point deduction if incorrect.
  * Figures are correctly used? Max. 2 point deduction if incorrect.
  * Equations are part of a sentence? Max. 2 point deduction if incorrect.
  * Tables and figures have captions? Max. 2 point deduction if incorrect.
  * Equations, tables and figures have numbers? Max. 2 point deduction if incorrect.
  * The author uses the active tense as much as possible throughout the writing assignment? Max. 5 point deduction if incorrect.
  * The reasoning is logical and does not contain any flaws?
  * The reasoning does not have serious omissions?
  * Simulation results are included in the main text?
  * The writing assignment explains the simulations in sufficient detail? Max. 5 point deduction if incorrect.
- **conclusions and discussion**: 0–15 points: the writing assignment draws correct and important conclusions and discusses possible shortcomings and suggestions for further research?
- **references**: 0–5 points: the writing assignment contains relevant references?

6.3 Exams

- the main form of summative assessment are three exams. The first exam will cover the material of Chs. 1 and 2. The second exam will cover the material of Chs. 1, 2, 3 and 4. The third and final exam will cover all course material. This implies that the material of Ch. 2 will come back in all three exams. The material of Chs. 3 and 4 will come back in the second and in the final exam.
- Each of the exams contributes 25% to the final grade for this course.
- Sec. 7 Course Schedule shows the exact dates of the exams.

– Exam questions are of the same level of difficulty as the homework questions. Answers to problems on the exam are assessed as follows:

(3 out of 3)	**(2 out of 3)**	**(1 out of 3)**	**(0 out of 3)**
The given answer is completely correct; student demonstrates clear reasoning and shows intermediate steps in sufficient detail; answer follows logically from the available information. In case the reasoning is crystal clear, but the final answer is derived from incorrect information, *e.g.* student used incorrect results from previous parts, then the student will earn full credit for the problem.	The given answer is not completely correct; student demonstrates sufficient understanding and shows intermediate steps in some detail, but with some minor mistakes. There are minor flaws in the reasoning.	The given answer is wrong, but the student demonstrates some understanding how the final answer to the problem can be found, but the student does not master that approach. There are serious short comings in the answer and in the reasoning.	The answer is completely wrong and the student does not demonstrate sufficient understanding how to answer the given problem, or the correct answer is given, but not supported by any reasoning. The problem is not at all answered.

7 Course schedule

The course schedule may be subject to small changes; as a result of class dynamics, some topics will be discussed earlier *cq.* later compared to the schedule below.

Time	**Topics**	**Course material**	**Assignments & assessment**
Week 1 MON, 26-08	Introduction to course; Set theory, Mappings, $A(S)$ set of 1-1 mappings of S onto itself	Sec. 1.1 - 1.4	Homework series 1 (Ch. 1) out
Week 1 THU, 29-08	Integers, Induction	Sec. 1.5, 1.6	
Week 2 MON, 02-09	Definitions and Examples of Groups, Some Simple Remarks	Sec. 2.1 - 2.2	
Week 2 THU, 05-09	Subgroups	Sec. 2.3	Submit Homework series 1; Homework series 2 (Ch. 2, sec. 1-4) out

Week 3 MON, 09-09	Lagrange's Theorem (1 out of 2)	Sec. 2.4	
Week 3 THU, 12-09	Lagrange's Theorem (2 out of 2)	Sec. 2.4	

Week 4 MON, 16-09	Homomorphisms and Normal Subgroups (1 out of 2)	Sec. 2.5	
Week 4 THU, 19-09	Homomorphisms and Normal Subgroups (2 out of 2)	Sec. 2.5	Submit Homework series 2; Homework series 3 (Ch. 2, sec. 5-11) out

Week 5 MON, 23-09	Factor Groups (1 out of 2)	Sec. 2.6	
Week 5 THU, 26-09	Factor Groups (2 out of 2)	Sec. 2.6	

Week 6 MON, 30-09	Homomorphism Theorems	Sec. 2.7	
Week 6 THU, 03-10	Cauchy's Theorem	Sec. 2.8	

Week 7 MON, 07-10	Direct Products and Finite Abelian Groups	Sec. 2.9 - 2.10	Writing assignment out
Week 7 THU, 10-10	Conjugacy and Sylow's Theorem	Sec 2.11	

14-10/18-10	Fall Break

Week 8 MON, 21-10	REVIEW CHS. 1–2		Submit Homework series 3; Homework series 4 (Ch. 3) out
Week 8 THU, 24-10	EXAM 1: GROUPS	CHS. 1–2	

Week 9 MON, 28-10	Symmetric Group	Ch. 3	Homework series 5 out
Week 9 THU, 31-10	NO CLASS, MODERATION		Submit Writing Assignment

Week 10 MON, 4-11	Definitions and Examples, Simple Results	Sec. 4.1 - 4.2	
Week 10 THU, 07-11	Ideals, Homomorphisms and Quotient Rings, Maximal Ideals	Sec. 4.3 - 4.4	Submit Homework series 4; Homework series 5 (Ch. 4) out

Week 11 MON, 11-11	Polynomial Rings	Sec. 4.5	
Week 11 THU, 14-11	Polynomials over the Rationals	Sec. 4.6	

Week 12 MON, 18-11	REVIEW CHS. 1–4		Submit Homework series 5; Homework series 6 (Ch. 5) out
Week 12 THU, 21-11	EXAM 2: GROUPS & RINGS	CHS. 1–4	
Week 13 MON, 25-11	Examples of Fields Field Extensions	Sec. 5.1 - 5.3	
Week 13 THU, 28-11	Finite extensions	Sec. 5.4	
Week 14 MON, 02-12	Constructibility	Sec. 5.5	
Week 14 THU, 05-12	Roots of polynomials	Sec. 5.6	
Week 15 MON, 09-12	REVIEW CHS. 1–5		Submit Homework series 6
Week 15 THU, 12-12	EXAM 3: GROUPS, RINGS & FIELDS	CHS. 1–5	

8 Student learning outcomes

This section list a number of learning outcomes. Note that this list is not complete; it is indicative for the course. Note that the student is expected to demonstrate his / her understanding of the material by **applying** his / her understanding to problems. Basically speaking, everywhere where it says 'understands', one should read 'understands and is able to demonstrate this understanding by applying this knowledge to a problem related to this'.
After following this course, the student

- (Ch. 1) understands the concept of a mapping, understands the concepts surjective, injective and bijective, is able to define a mapping and to prove if a mapping is surjective, injective, bijective.
- (Ch. 1) is able to apply proofs by mathematical induction in simple cases and can understand proofs involving mathematical induction as used in the text.
- (Ch. 2) understands what a group is, is able to verify if a given set and operation form a group, is able to find subgroups in a group, understands what an abelian group is, understands what a cyclic group is.
- (Ch. 2) understands what an equivalence relation is and is able to prove if a relation is an equivalence relation, understands that an equivalence relation partitions a group into equivalence classes.
- (Ch. 2) understands Lagrange's Theorem.
- (Ch. 2) understands what a homomorphism is and is able to prove if a given mapping is a homomorphism, monomorphism, isomorphism or automorphism, understands what the kernel of a mapping is.
- (Ch. 2) understands what left and right cosets are, understands what normal subgroups are.

- (Ch. 2) understands what factor groups are and is able to define factor groups given a group and a normal subgroup.
- (Ch. 2) understands the homomorphism theorems and is able to apply the homomorphism theorems.
- (Ch. 2) understands Cauchy's Theorem for abelian groups and nonabelian groups.
- (Ch. 2) understands the concept of direct products and indirect products.
- (Ch. 2) understands the fundamental theorem on finite abelian groups.
- (Ch. 2) understands the three parts of Sylow's theorem and is able to apply Sylow's theorem to groups of small order.
- (Ch. 3) understands the concept of the Symmetric Group, understands cycle decomposition, understands odd and even permutations, understands the concept of the alternating group.
- (Ch. 4) understands the concept of a ring and a subring, is able to prove if a set under addition and multiplication forms a ring.
- (Ch. 4) understands the concept of an ideal and quotient rings, understands the homomorphism theorems for rings.
- (Ch. 4) understands the concept of maximal ideal.
- (Ch. 4) understands the concept of polynomial rings and is able to apply the Eisenstein criterion.
- (Ch. 5) understands the concept of a field and is able to prove if a set form a fields under addition and multiplication.
- (Ch. 5) understands the concept of the characteristic of a field.
- (Ch. 5) understands the concept of vector spaces.
- (Ch. 5) understands the concept of field extensions and is able to find field extensions for algebraic numbers.
- (Ch. 5) understands the concept of constructibility using straight edge and compass.